

ECE212/212s Lecture Notes

Lecture: Appendix A

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Number systems

Today we will cover Number Systems. You will learn how to count all over again. How many people know how to count? How many people know how to count in another number system?

0.1 Positional Notation

Positional means that a digit's place determines its weight.

Base 10 has 10 digits 0,1,2,3,4,5,6,7,8,9.

Base 3 has 3 digits 0,1,2.

Base 2 has 2 digits 0,1.

Base 8 has 8 digits 0,1,2,3,4,5,6,7.

Base 16 has 16 digits 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F.

$$\begin{aligned} 154_{10} &= 1X100 + 5X10 + 4 \\ &= 1X10^2 + 5X10^1 + 4X10^0 \end{aligned}$$

$$\begin{aligned} 12201_3 &= 1X3^4 + 2X3^3 + 2X3^2 + 0X3^1 + 1X3^0 \\ &= 81 + 54 + 18 + 0 + 1 \\ &= 154 \end{aligned}$$

Decimal	Binary	Octal	Hexadecimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13
20	10100	24	14
21	10101	25	15
22	10110	26	16
23	10111	27	17
24	11000	30	18
25	11001	31	19
26	11010	32	1A
27	11011	33	1B
28	11100	34	1C
29	11101	35	1D
30	11110	36	1E
31	11111	37	1F

Conversion from Binary,Octal,Hex to Decimal

$$\begin{aligned}
10011010_2 &= 1X2^7 + 0X2^6 + 0X2^5 + 1X2^4 + 1X2^3 + 0X2^2 + 1X2^1 + 0X2^0 \\
&= 1X128 + 0X64 + 0X32 + 1X16 + 1X8 + 0X4 + 1X2 + 0X1 \\
&= 128 + 16 + 8 + 2 \\
&= 154_{10}
\end{aligned}$$

$$232_8 = 2X8^2 + 3X8^1 + 2X8^0$$

$$\begin{aligned}
&= 2X64 + 3X8 + 2X1 \\
&= 128 + 24 + 2 \\
&= 154_{10}
\end{aligned}$$

$$\begin{aligned}
9A_{16} &= 9X16^1 + 10X16^0 \\
&= 9X16 + 10X1 \\
&= 144 + 10 \\
&= 154_{10}
\end{aligned}$$

Conversion from Binary to Octal

Use Bit Grouping. Grouping index is 3. 3 binary digits = 1 octal digit.

$$10011010_2 = 010 \ 011 \ 010 = 232_8$$

Conversion from Binary to Hexadecimal

Use Bit Grouping. Grouping index is 4.

$$4 \text{ binary digits} = 1 \text{ hexadecimal digit. } 10011010_2 = 1001 \ 1010 = 9A_{16}$$

Conversion from Decimal to Binary

Use successive division. $154_{10} = 10011010_2$

$$\begin{aligned}
154 \div 2 &= 77R0 \text{ (LSB)} \\
77 \div 2 &= 38R1 \\
38 \div 2 &= 19R0 \\
19 \div 2 &= 9R1 \\
9 \div 2 &= 4R1 \\
4 \div 2 &= 2R0 \\
2 \div 2 &= 1R0 \\
1 \div 2 &= 0R1 \text{ (MSB)}
\end{aligned}$$

Conversion from Octal to Binary

Use Reverse Bit Grouping. Grouping index is 3. 1 octal digit = 3 binary digits.

$$234_8 = 010 \ 011 \ 100_2 = 010011100_2$$

Conversion from Octal to Hex

First convert from octal to binary (RBG) and then from binary to hex (BG).

$$234_8 = 010\ 011\ 100_2 = 010011100_2$$

$$0\ 1001\ 1100_2 = 09C_{16}$$

Conversion from Hex to Octal

First convert from hexadecimal to binary (RBG) and then from binary to octal(BG).

$$F A F B_{16} = 1111\ 1010\ 1111\ 1011_2$$

$$1\ 111\ 101\ 011\ 111\ 011_2 = 175373_8$$

The Quaternary System - Base 4 To convert from Base 4 to Base 2, we can use reverse bit grouping with the grouping index of 2.

$$232_4 = 10\ 11\ 10_2$$

Group Exercise

Cover the number 1012_3 to base 7.

We could use successive division using 7 as the denominator, but we would have to divide in base 3. How many people can divide by 7 in base 3? How many can do it quickly?

Therefore, to convert from base X to base Y, we can first use PN to convert from base X to base 10, then we can convert from base 10 to base Y using SD.

BASE X $\rightarrow$ BASE Y (SD - We must divide by Y in BASE X)

Instead:

BASE X $\rightarrow$ BASE 10 (PN)

BASE 10 $\rightarrow$ BASE Y (SD)

$$\begin{aligned} 1012_3 &= 1X3^3 + 0X3^2 + 1X3^1 + 2X3^0 \\ &= 1X27 + 0X9 + 1X3 + 2X1 \\ &= 27 + 0 + 3 + 2 \\ &= 32 \end{aligned}$$

$$\begin{aligned} 32_{10} \quad 32/7 &= 4R4 \\ 4/7 &= 0R4 \end{aligned}$$

The following table illustrates the methods used to convert numbers in the number system of the row into the number system of the column.

	Decimal	Binary	Octal	Hexadecimal
Decimal	-	SD	SD	SD
Binary	PN	-	BG	BG
Octal	PN	RBG	-	RBG,BG
Hexadecimal	PN	RBG	RBG,BG	-

SD - Successive Division

PN - Positional Notation

BG - Bit Grouping

RBG - Reverse Bit Grouping

Addition in Positional Notation

Use $95_{10} + 16_{10} = ?$

(Remember to illustrate the carry process.)

$$\begin{aligned}
 95_{10} &= 9 \times 10^1 + 5 \times 10^0 \\
 16_{10} &= 1 \times 10^1 + 6 \times 10^0 \\
 111_{10} &= 10 \times 10^1 + 11 \times 10^0 \\
 &= (10 + 1) \times 10^1 + 1 \times 10^0 \\
 &= 11 \times 10^1 + 1 \times 10^0 \\
 &= 1 \times 10^2 + 1 \times 10^1 + 1 \times 10^0
 \end{aligned}$$

Addition in Binary

$$0+0=0$$

$$0+1=1$$

$$1+0=1$$

$$1+1=10 \text{ (Zero Carry one)}$$

$$\begin{aligned}
 95_{10} &= 1011111 \\
 16_{10} &= 0010000 \\
 111_{10} &= 1101111
 \end{aligned}$$

Subtraction in Positional Notation

Use $95_{10} - 16_{10} = ??$

(Remember to illustrate the borrowing process.)

$$\begin{aligned} 95_{10} &= 9X10^1 + 5X10^0 = 8X10^1 + 15X10^0 \\ 16_{10} &= 1X10^1 + 6X10^0 = 1X10^1 + 6X10^0 \\ 79_{10} &= \quad \quad \quad = \quad \quad \quad 7X10^1 + 9X10^0 \end{aligned}$$

Subtraction in Binary

$$0-0=0$$

$$0-1=1 \text{ (Need to Borrow)}$$

$$1-0=1$$

$$1-1=0$$

$$95_{10} = 1011111$$

$$16_{10} = 0010000$$

$$79_{10} = 1001111$$

$$4_{10} = 100$$

$$1_{10} = 001$$

$$3_{10} = 011$$