

Name: ANSWERS
Mod & Sim Quiz #1, September 23, 2004, Thursday

A sample space consists of the three points

$$\Omega = \{w_1, w_2, w_3\}$$

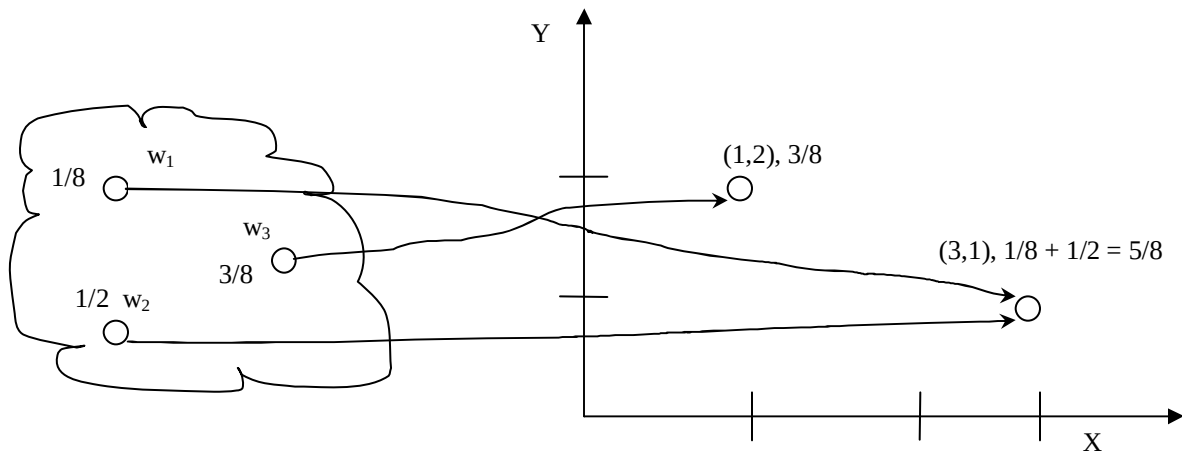
and the probability of the simple events are

$$P[\{w_1\}] = \frac{1}{8}, \quad P[\{w_2\}] = \frac{1}{2}, \quad P[\{w_3\}] = \frac{3}{8}.$$

Define random variables X and Y by:

$$\begin{array}{lll} X(w_1) = 3, & X(w_2) = 3 & X(w_3) = 1 \\ Y(w_1) = 1 & Y(w_2) = 1 & Y(w_3) = 2 \end{array}$$

Draw the sample space and Euclidean space mapping:



Fill in the probability distribution tables:

Probability Distribution for X

$X(w_i)$	$P[X(w_i)]$
1	$3/8$
3	$1/8 + 1/2 = 5/8$

Probability Distribution for Y

$Y(w_i)$	$P[Y(w_i)]$
1	$1/8 + 1/2 = 5/8$
2	$3/8$

Give the Distribution Functions:

$$F_X(x) = \begin{cases} 0, & -\infty < x \leq 1 \\ 3/8, & 1 \leq x < 3 \\ 1, & 3 \leq x < \infty \end{cases}$$

$$F_Y(y) = \begin{cases} 0, & -\infty < y \leq 1 \\ 5/8, & 1 \leq y < 2 \\ 1, & 2 \leq y < \infty \end{cases}$$

Find the conditional probabilities $P[y_j | x_i]$ for all possible pairs $X = x_i, Y = y_j$.

$$P[y_j | x_i] = \frac{P[x_i, y_j]}{P[x_i]}$$

$$P[1|1] = \frac{P[1,1]}{P[x=1]} = \frac{0}{3/8} = 0 \quad P[1|3] = \frac{P[3,1]}{P[x=3]} = \frac{5/8}{5/8} = 1$$

$$P[2|1] = \frac{P[1,2]}{P[x=1]} = \frac{3/8}{3/8} = 1 \quad P[2|3] = \frac{P[3,2]}{P[x=3]} = \frac{0}{5/8} = 0$$

Are X and Y statistically independent? Justify your answer.

No! X and Y are not statistically independent. To be statistically independent:

$P[y_j | x_i] = P[y_i]$, or $P[x_i, y_i] = P[x_i]P[y_i]$ must hold. Above we see that:

$$P[1|1] = 0, \quad P[x=1]P[y=1] = (3/8)(5/8) = 15/64, \text{ which are not equal!}$$