

Homework Assignment 3 SOLUTIONS

1. (*Canonical Forms*) A Boolean expression for the function $F(A, B, C, D)$ is given by

$$F(A, B, C, D) = (\overline{A} + \overline{B} + D) (A + \overline{B} + C) (\overline{A} + B) (B + \overline{D})$$

- a. Use DeMorgan's theorem to compute the complement of $F(A, B, C, D)$.

Solution

$$\begin{aligned} \overline{F(A, B, C, D)} &= \overline{(\overline{A} + \overline{B} + D)(A + \overline{B} + C)(\overline{A} + B)(B + \overline{D})} \\ &= \overline{(\overline{A} + \overline{B} + D)} + \overline{(A + \overline{B} + C)} + \overline{(\overline{A} + B)} + \overline{(B + \overline{D})} \\ &= A B \overline{D} + \overline{A} B \overline{C} + A \overline{B} + \overline{B} D \end{aligned}$$

- b. Write $F(A, B, C, D)$ as a Boolean expression in canonical maxterm form. **Solution**

$$\begin{aligned} F(A, B, C, D) &= (\overline{A} + \overline{B} + D)(A + \overline{B} + C)(\overline{A} + B)(B + \overline{D}) \\ &= (\overline{A} + \overline{B} + D + (\overline{C} C)) \\ &\quad (A + \overline{B} + C + (\overline{D} D)) \\ &\quad (\overline{A} + B + (\overline{C} C) + (\overline{D} D)) \\ &\quad (B + \overline{D} + (\overline{A} A) + (\overline{C} C)) \text{ Complementarity} \\ &= (\overline{A} + \overline{B} + \overline{C} + D)(\overline{A} + \overline{B} + C + D) \\ &\quad (A + \overline{B} + C + \overline{D})(A + \overline{B} + C + D) \\ &\quad (\overline{A} + B + \overline{C} + \overline{D})(\overline{A} + B + \overline{C} + D)(\overline{A} + B + C + \overline{D})(\overline{A} + B + C + D) \\ &\quad (\overline{A} + B + \overline{C} + \overline{D})(\overline{A} + B + C + \overline{D})(A + B + \overline{C} + \overline{D})(A + B + C + \overline{D}) \text{ Distributive} \\ &= (\overline{A} + \overline{B} + \overline{C} + D)(\overline{A} + \overline{B} + C + D) \\ &\quad (A + \overline{B} + C + \overline{D})(A + \overline{B} + C + D) \\ &\quad (\overline{A} + B + \overline{C} + \overline{D})(\overline{A} + B + \overline{C} + D)(\overline{A} + B + C + \overline{D})(\overline{A} + B + C + D) \\ &\quad (A + B + \overline{C} + \overline{D})(A + B + C + \overline{D}) \text{ Idempotent} \end{aligned}$$

- c. Write $F(A, B, C, D)$ as a product of sums using "big M" notation.

Solution

From the previous problem, we get:

$$\begin{aligned} F(A, B, C, D) &= (\overline{A} + \overline{B} + \overline{C} + D)(\overline{A} + \overline{B} + C + D) \\ &\quad (A + \overline{B} + C + \overline{D})(A + \overline{B} + C + D) \\ &\quad (\overline{A} + B + \overline{C} + \overline{D})(\overline{A} + B + \overline{C} + D)(\overline{A} + B + C + \overline{D})(\overline{A} + B + C + D) \\ &\quad (A + B + \overline{C} + \overline{D})(A + B + C + \overline{D}) \\ &= \prod M(14, 12, 5, 4, 11, 10, 9, 8, 3, 1) \\ &= \prod M(1, 3, 4, 5, 8, 9, 10, 11, 12, 14) \end{aligned}$$

- d. Write the **complement** of $F(A, B, C, D)$ using “little m” notation.

Solution

$$\begin{aligned}
 \overline{F(A, B, C, D)} &= A B \overline{D} + \overline{A} B \overline{C} + A \overline{B} + \overline{B} D \\
 &= A B \overline{D}(\overline{C} + C) + \\
 &\quad \overline{A} B \overline{C}(\overline{D} + D) + \\
 &\quad A \overline{B}(\overline{C} + C)(\overline{D} + D) + \\
 &\quad \overline{B} D(\overline{A} + A)(\overline{C} + C) \\
 &= A B \overline{C} \overline{D} + A B C \overline{D} + \\
 &\quad \overline{A} B \overline{C} \overline{D} + \overline{A} B \overline{C} D + \\
 &\quad A \overline{B} \overline{C} \overline{D} + A \overline{B} \overline{C} D + A \overline{B} C \overline{D} + A \overline{B} C D + \\
 &\quad \overline{A} \overline{B} \overline{C} \overline{D} + \overline{A} \overline{B} \overline{C} D + \overline{A} \overline{B} C \overline{D} + \overline{A} \overline{B} C D \\
 &= A B \overline{C} \overline{D} + A B C \overline{D} + \\
 &\quad \overline{A} B \overline{C} \overline{D} + \overline{A} B \overline{C} D + \\
 &\quad A \overline{B} \overline{C} \overline{D} + A \overline{B} \overline{C} D + A \overline{B} C \overline{D} + A \overline{B} C D + \\
 &\quad \overline{A} \overline{B} \overline{C} \overline{D} + \overline{A} \overline{B} \overline{C} D \\
 &= \sum m(12, 14, 4, 5, 8, 9, 10, 11, 1, 3) \\
 &= \sum m(1, 3, 4, 5, 8, 9, 10, 11, 12, 14)
 \end{aligned}$$

2. (*Canonical Forms*) Derive a *product of sums* expression for the ALARM output found in problem ?? and express it in the “big M” notation.

Solution From the truth table, we write down all maxterms based on the values of the off-set (zeroes) of the function. We get:

$$ALARM = \prod M(0, 1, 2, 3, 4, 5, 6, 7, 11, 12, 13, 14, 15)$$

3. (*Laws and Theorems of Boolean Algebra*) For the given Boolean expression prove the following.

$$A \overline{B} + \overline{A} C + BC = A \overline{B} + C$$

- (a) Prove that the Boolean expression is true using the truth tables.

Solution

A	B	C	$\overline{A}$	$\overline{B}$	$A \overline{B}$	$\overline{A} C$	BC	$A \overline{B} + \overline{A} C + BC$	$\overline{A} B + C$
0	0	0	1	1	0	0	0	0	0
0	0	1	1	1	0	1	0	1	1
0	1	0	1	0	0	0	0	0	0
0	1	1	1	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1
1	0	1	0	1	1	0	0	1	1
1	1	0	0	0	0	0	0	0	0
1	1	1	0	0	0	0	1	1	1

Since the last two columns of the truth table are equivalent, the given Boolean functions are equivalent.

- (b) Prove that the Boolean expression is true using the laws and theorems of Boolean algebra found in the text on page 51. Make sure that each step uses only one law and write the law used beside each step.

Solution

First, we get the left hand side into canonical form.

$$\begin{aligned}
 A \overline{B} + \overline{A} C + BC &= A \overline{B}(\overline{C} + C) \\
 &\quad + \overline{A} C(\overline{B} + B) \\
 &\quad + BC(\overline{A} + A) \text{ Complimentarity} \\
 &= A \overline{B} \overline{C} + A \overline{B} C \\
 &\quad + \overline{A} \overline{B} C + \overline{A} B C \\
 &\quad + \overline{A} B C + A B C \text{ Distributive} \\
 &= A \overline{B} \overline{C} + A \overline{B} C \\
 &\quad + \overline{A} \overline{B} C + \overline{A} B C \\
 &\quad + A B C \text{ Idempotent} \\
 &= \sum m(1, 3, 4, 5, 7)
 \end{aligned}$$

Next, we get the left hand side into canonical form.

$$\begin{aligned}
 A \overline{B} + C &= A \overline{B}(\overline{C} + C) \\
 &\quad + C(\overline{A} + A)(\overline{B} + B) \text{ Complimentarity} \\
 &= A \overline{B} \overline{C} + A \overline{B} C \\
 &\quad + \overline{A} \overline{B} C + \overline{A} B C + A \overline{B} C + A B C + \text{Distributive} \\
 &= A \overline{B} \overline{C} + A \overline{B} C \\
 &\quad + \overline{A} \overline{B} C + \overline{A} B C + A B C + \text{Idempotent} \\
 &= \sum m(1, 3, 4, 5, 7)
 \end{aligned}$$

Therefore the left hand side is equal to the right hand side since they both are the same in canonical form (transitive property of equality).

$$A \overline{B} + \overline{A} C + BC = A \overline{B} + C$$

4. (*Boolean Simplification*) Consider the Boolean function

$$F(A, B, C, D, E) = \sum m(0, 2, 7, 8, 9, 10, 11, 12, 14, 16, 18, 24, 26) + \sum d(1, 3, 5)$$

Use a Karnaugh Map to find a minimized **sum of products** (SOP) expression for $F(A, B, C, D, E)$.

Solution

In your KMAP, you should have circled the following prime implicants:

- Prime Implicant 1: (d_1, d_3, d_5, m_7) results in the term $\overline{A} \overline{B} E$
- Prime Implicant 2: $(m_0, m_2, m_8, m_{10}, m_{16}, m_{18}, m_{24}, m_{26})$ results in the term $\overline{C} \overline{E}$
- Prime Implicant 3: $(m_8, m_{10}, m_{12}, m_{14})$ results in the term $\overline{A} B \overline{E}$
- Prime Implicant 4: $(m_0, d_1, m_2, d_3, m_8, m_9, m_{10}, m_{11})$ results in the term $\overline{A} \overline{C}$

Therefore, the function is:

$$F(A, B, C, D, E) = \overline{A} \overline{B} E + \overline{C} \overline{E} + \overline{A} B \overline{E} + \overline{A} \overline{C}$$

5. (*Boolean Simplification*) Use espresso to work Problem 4 above and compare your results with your results using Karnaugh maps. See *espresso_intro.ps* in the *ece212_info* locker under the directory “/ncsu/ece212_info/general/ps” and the examples under the directory “/ncsu/ece212_info/examples/chap2/ps”.

Solution

The espresso input file is shown below:

file: fivevar.esp

```
.i 5
.o 1
.p 32
.ilb A B C D E
.ob F
00000 1
00001 -
00010 1
00011 -
00100 0
00101 -
00110 0
00111 1
01000 1
01001 1
01010 1
01011 1
01100 1
01101 0
01110 1
01111 0
10000 1
10001 0
10010 1
10011 0
10100 0
10101 0
```

```

10110 0
10111 0
11000 1
11001 0
11010 1
11011 0
11100 0
11101 0
11110 0
11111 0
.e

```

The espresso output file is shown below:
file: fivevar.min

```

.i 5
.o 1
.ilb A B C D E
.ob F
.p 4
00--1 1
--0-0 1
01--0 1
0-0-- 1
.e

```

From the espresso file, we read the equation:

$$F = \overline{A} \overline{B} E + \overline{C} \overline{E} + \overline{A} B \overline{E} + \overline{A} \overline{C}$$