

Question 1	Question 2	Question 3	Question 4	Question 5	Total Points

ECE212 and ECE212s
Fundamentals of Logic Design
Fall 1996
Tuesday, September 24, 1996
Examination 1 - Solutions

Name (Please Print)

Social Security Number

This is a 75-minute open-book test. Please put away all papers except the exam itself and your text. You can use the back of the pages for work space. However, make sure that your answers are placed in the space provided for each problem. Clearly indicate your answers. Answer *all* questions and remember to check your answers carefully. Be sure to check the point value of each question before beginning. For partial credit, show your work, especially where the answer is just a number.
(a)

Here is a list of questions and subjects.

Question	Topic	Page
1.	Number Conversion	1
2.	Logic Simplification using Truth Tables and Boolean Algebra	2
3.	Logic Implementation using NAND gates only	5
4.	Two Level Canonical Forms	6
5.	Two Level Combinational Circuit Design	7

1. (*Number Conversions*) - 20 points. Complete the following conversion problems. Don't forget to show your work!!! Correct answers with no work will receive **NO** credit.

- a. Convert the number $FAD9_{16}$ to decimal. (5 points)

$$\begin{aligned} FAD9_{16} &= (15 \times 16^3) + (10 \times 16^2) + (13 \times 16^1) \\ &= 64217_{10} \end{aligned}$$

- b. Convert the number 334_{10} to octal. (5 points)

Solution

$$334_{10} = ?_8$$

$$\begin{aligned} 334 \div 8 &= 41 \text{ } R \text{ } 6 \\ 41 \div 8 &= 5 \text{ } R \text{ } 1 \\ 5 \div 8 &= 0 \text{ } R \text{ } 5 \end{aligned}$$

$$334_{10} = 516_8$$

- c. Perform the following addition using base 16 arithmetic. (5 points)

$$47EF + 3244$$

$$\begin{array}{r} 0110_{16} \quad \text{Carry} \\ 47EF_{16} \quad \text{Operand} \\ + 3244_{16} \quad \text{Operand} \\ \hline 7A33_{16} \quad \text{Result} \end{array}$$

- d. Perform the following subtraction using 8-bit two's complement numbers. (5 points)
 $-120_{10} - 9_{10}$. Show both operands and the result in two's complement. Denote if overflow occurs. $-120_{10} - 9_{10} = -120 + -9$

$$\begin{array}{r} 10001000_2 \\ + 11110111_2 \quad \text{Overflow} \\ \hline 01111111_2 \end{array}$$

Name:

2

2. (*Boolean Simplification - 20 points.*) Prove that the following Boolean expressions are true.

a. (10 points) Prove that the following Boolean expression is true using truth tables.

$$A + B + C = (A + \overline{A}B + C + \overline{D})(A + \overline{A}B + C + D)$$

A	B	C	D	(A + B + C)
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Name:

3

A	B	C	D	$\overline{A}$	$\overline{D}$	$\overline{A} B$	$(A + \overline{A} B + C + \overline{D})$	$(A + \overline{A} B + C + D)$	$(A + \overline{A} B + C + \overline{D})$ $(A + \overline{A} B + C + D)$
0	0	0	0	1	1	0	1	0	0
0	0	0	1	1	0	0	0	1	0
0	0	1	0	1	1	0	1	1	1
0	0	1	1	1	0	0	1	1	1
0	1	0	0	1	1	1	1	1	1
0	1	0	1	1	0	1	1	1	1
0	1	1	0	1	1	1	1	1	1
0	1	1	1	1	0	1	1	1	1
1	0	0	0	0	1	0	1	1	1
1	0	0	1	0	0	0	1	1	1
1	0	1	0	0	1	0	1	1	1
1	0	1	1	0	0	0	1	1	1
1	1	0	0	0	1	0	1	1	1
1	1	0	1	0	0	0	1	1	1
1	1	1	0	0	1	0	1	1	1
1	1	1	1	0	0	0	1	1	1

Since the last column in both tables is equivalent, we have shown that:

$$A + B + C = (A + \overline{A} B + C + \overline{D}) (A + \overline{A} B + C + D)$$

- b. (10 points) Prove that the following Boolean expression is true using laws and theorems of Boolean algebra.

$$A + B + C = (A + \overline{A}B + C + \overline{D})(A + \overline{A}B + C + D)$$

First, we put the left hand side in canonical POS form.

$$\begin{aligned} A + B + C &= (A + B + C + (\overline{D}D)) && \text{Complementarity} \\ &= (A + B + C + \overline{D})(A + B + C + D) && \text{Distributive} \\ &= M_1 + M_0 && \text{Rewriting maxterms} \\ &= \prod M(0,1) && \text{Rewriting in big m notation} \end{aligned}$$

Next, we put the right hand side in canonical POS form.

$$\begin{aligned} &(A + \overline{A}B + C + \overline{D})(A + \overline{A}B + C + D) \\ &= (A(\overline{B} + B) + \overline{A}B + C + \overline{D})(A(\overline{B} + B) + \overline{A}B + C + D) && \text{Complementarity} \\ &= (A\overline{B} + AB + \overline{A}B + C + \overline{D})(A\overline{B} + AB + \overline{A}B + C + D) && \text{Distributive} \\ &= (A\overline{B} + AB + AB + \overline{A}B + C + \overline{D})(A\overline{B} + AB + AB + \overline{A}B + C + D) && \text{Idempotent} \\ &= (A(\overline{B} + B) + (A + \overline{A})B + C + \overline{D})(A(\overline{B} + B) + (A + \overline{A})B + C + D) && \text{Distributive} \\ &= (A + B + C + \overline{D})(A + B + C + D) && \text{Complementarity} \\ &= M_1 + M_0 && \text{Rewriting maxterms} \\ &= \prod M(0,1) && \text{Rewriting in big m notation} \end{aligned}$$

Name:

5

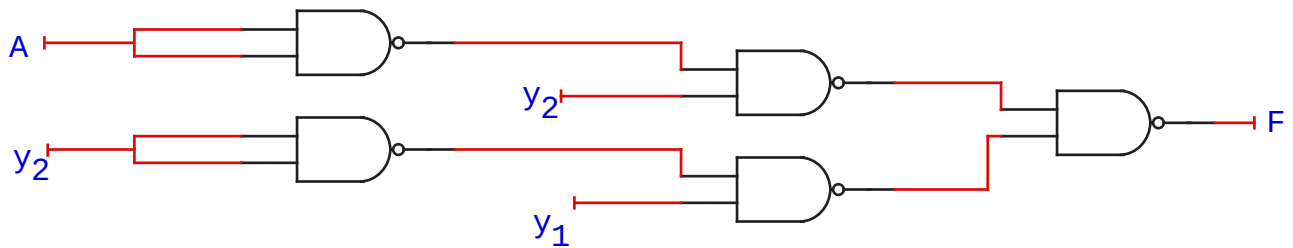
3. (Implementation with NAND gates - 20 points.) Implement the following Boolean expression using NAND gates only. Only the true forms of all variables are available as inputs to the circuit. (i.e. $\bar{A}$ is not available as an input.) Use a KMAP to simplify the circuit first.

$$F(A, y_2, y_1) = \sum m(1, 2, 3, 5)$$

$$F(A, y_1, y_2) = \text{sum}(1, 2, 3, 5)$$

$$F(A, y_2, y_1) = A'y_2 + y_2'y_1$$

		Ay ₂			
		00	01	11	10
y ₁	0	0 0	1 2	0 6	0 4
	1	1 1	1 3	0 7	1 5



4. (*Canonical Forms - 20 points.*) A Boolean expression for the function $F(A, B, C, D)$ is given by

$$F(A, B, C) = (A \oplus B)C$$

Remember that $X \oplus Y = \overline{X}Y + X\overline{Y}$.

- a. Write the function $F(A, B, C)$ in canonical SOP form using little m notation.

$$\begin{aligned} F(A, B, C) &= (A \oplus B)C \\ &= (\overline{A}B + A\overline{B})C \\ &= \overline{A}BC + A\overline{B}C \\ &= m_3 + m_5 \\ &= \sum m(3, 5) \end{aligned}$$

- b. Write the function $F(A, B, C)$ in canonical POS form using big M notation.

$$\begin{aligned} F(A, B, C) &= \sum m(3, 5) \\ &= \prod M(0, 1, 2, 4, 6, 7) \end{aligned}$$

- c. Write the **complement** of the function, $\overline{F(A, B, C)}$, in canonical SOP form using little m notation.

$$\overline{F(A, B, C)} = \sum m(0, 1, 2, 4, 6, 7)$$

- d. Write the **complement** of the function, $\overline{F(A, B, C)}$, in canonical POS form using big M notation.

$$\overline{F(A, B, C)} = \prod M(3, 5)$$

5. (*Two Level Combinational Circuit Design*) - 20 points. Consider a binary decrement by two circuit for four bit numbers. The function of this circuit is to decrement the input number by two.

- a. Complete the truth table for the decrementer. Remember that the inputs with decimal value greater than 9 will never be seen by this circuit. (5 points)

Example 4-bit binary decrement by two

A	B	C	D	W	X	Y	Z
0	0	0	0	1	0	0	0
0	0	0	1	1	0	0	1
0	0	1	0	0	0	0	0
0	0	1	1	0	0	0	1
0	1	0	0	0	0	1	0
0	1	0	1	0	0	1	1
0	1	1	0	0	1	0	0
0	1	1	1	0	1	0	1
1	0	0	0	0	1	1	0
1	0	0	1	0	1	1	1
1	0	1	0	X	X	X	X
1	0	1	1	X	X	X	X
1	1	0	0	X	X	X	X
1	1	0	1	X	X	X	X
1	1	1	0	X	X	X	X
1	1	1	1	X	X	X	X

- b. Represent the output functions W and Y in sum of products form using little m notation. (5 points)

$$W = \sum m(0, 1) + \sum d(10, 11, 12, 13, 14, 15)$$

$$Y = \sum m(4, 5, 8, 9) + \sum d(10, 11, 12, 13, 14, 15)$$

Name:

8

- c. Use the provided Karnaugh maps to find a minimized sum of products form for W and Y. (5 points)

		A B			
		00	01	11	10
C D	00	1 ₀	0 ₄	X ₁₂	0 ₈
	01	1 ₁	0 ₅	X ₁₃	0 ₉
	11	0 ₃	0 ₇	X ₁₅	X ₁₁
	10	0 ₂	0 ₆	X ₁₄	X ₁₀

$$W = A' B' C'$$

		A B			
		00	01	11	10
C D	00	0 ₀	1 ₄	X ₁₂	1 ₈
	01	0 ₁	1 ₅	X ₁₃	1 ₉
	11	0 ₃	0 ₇	X ₁₅	X ₁₁
	10	0 ₂	0 ₆	X ₁₄	X ₁₀

$$Y = B C' + A$$

d. Draw the schematic for the decrementer output functions W and Y. (5 points)

